Grade 12
Pre-Calculus Mathematics
Achievement Test

Booklet 1

June 2014

Manitoba

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This resource will also be available on the Manitoba Education and Advanced Learning website at <www.edu.gov.mb.ca/k12/assess/archives/index.html>.

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Disponible en français.

Available in alternate formats upon request.

Grade 12 Pre-Calculus Mathematics Achievement Test

DESCRIPTION

Time: 3 hours

	Questions	Marks	Total Marks
Booklet 1*	16 Short-Answer	25	36
	3 Long-Answer	11	
Booklet 2	10 Multiple-Choice	10	54
	12 Short-Answer	18	
	8 Long-Answer	26	
Total			**90**

* The first 5 questions in *Booklet 1* require a calculator. 
You will have access to your calculator for the first 45 minutes of the test.

GENERAL DIRECTIONS

§ Read all instructions carefully.

§ The blank pages at the back of each booklet may be used as scrap paper, but must **not** be removed from the test booklet. No marks will be given for work done on these pages.

§ Note that diagrams and graphs provided in the test booklets may not be drawn to scale.

§ After 45 minutes, put away your calculator. Even though you may not have finished *Booklet 1*, *Booklet 2* will be distributed at this time. You may choose to continue working on *Booklet 1* or start working on *Booklet 2*, but you will no longer have access to your calculator.

Instructions

- There are 19 questions for a total of 36 marks.
- Calculators (scientific or graphing) are allowed for the first 45 minutes of the test.
- A calculator icon  appears next to the questions that require a calculator.
- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Graphing calculator solutions must include an explanation of how your final answer is obtained.
- Your solutions should be neat, organized, and clear.
- Some answers are to be given as decimal values. Rounding too early in your solution may result in an inaccurate final answer for which full marks will not be given.
- Express your answers as exact values or correct to 3 decimal places unless instructed otherwise.

Formula Sheet

$$s = \theta r$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$\cos 2\alpha = 1 - 2\sin^2\alpha$$

$$\cos 2\alpha = 2\cos^2\alpha - 1$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

$$\log_a(MN) = \log_a M + \log_a N$$

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$$

$$\log_a(M^n) = n\log_a M$$

$$P(n,r) \text{ or } {}_nP_r = \frac{n!}{(n-r)!}$$

$$C(n,r) \text{ or } {}_nC_r = \frac{n!}{r!(n-r)!}$$

$$t_{k+1} = {}_nC_k a^{n-k} b^k$$

Terminology Sheet

Some questions may contain directing words such as *explain*, *identify*, and *justify*. These words are explained below.

Evaluate: Find the numerical value.

Explain: Use words to provide the cause or reason for the response, or to render the response more clear and understandable.

Sketch the graph: Provide a detailed drawing with key features of the graph that includes a minimum of 2 coordinate points.

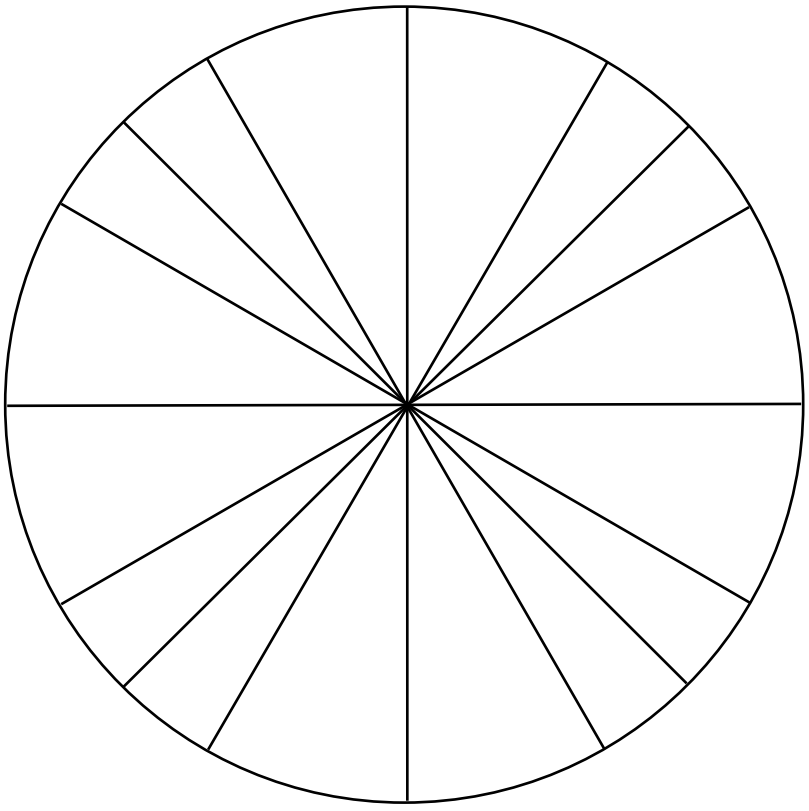
Identify / Indicate: Recognize and select the answer by stating or circling it.

Justify: Show reasons for or give facts that support a position by using mathematical computations, words, and/or diagrams.

Solve: Give a solution for a problem or determine the value(s) of a variable.

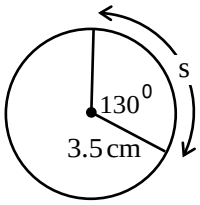
Verify: Establish the truth of a statement by substitution or comparison.

Unit Circle (can be used if needed)



Question 1  2 marks 101

Use the information in the diagram to determine the value of the arc length “s”.



Question 2  2 marks

Solve the following equation over the interval $[0,\ 2\pi)$.

$$\tan^2\theta + 2.8\tan\theta + 1.96 = 0$$

Use the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for $ax^2 + bx + c = 0$.

Question 3  3 marks 103

Determine how many monthly investments of $50 would have to be deposited into a savings account that pays 3% annual interest, compounded monthly, for the account's future value to be $50,000.

Use the formula:

$$\text{FV} = \frac{\text{R}\left[(1+i)^n - 1\right]}{i}$$

where FV = the future value
R = the investment amount
$i = \dfrac{\text{the annual interest rate}}{\text{the number of compounding periods per year}}$
n = the number of investments

Express your answer as a whole number.

Question 4  2 marks

104

There are 5 men and 4 women to be seated in a row.

How many arrangements are possible if two men must sit at the beginning of the row and two men must sit at the end of the row?

Question 5 

a) 3 marks b) 2 marks

105
106

a) In the binomial expansion of $\left(\frac{3}{x^2} - 4x^5\right)^8$, determine the 3rd term.

b) In the binomial expansion of $\left(\frac{3}{x^2} - 4x^5\right)^n$, the 6th term contains x^{25}.

Solve for n.

Note: A calculator is not required for the remaining test questions.

Question 6 — 1 mark — 107

Given the following two functions, $f(x) = \sqrt{x-1}$ and $g(x) = x^2 + 1$, evaluate $g(f(3))$.

Question 7 — 2 marks — 108

If θ terminates in quadrant II and $\csc\theta = \frac{3}{2}$, determine the exact value of $\tan\theta$.

Question 8

a) 1 mark b) 1 mark

109
110

a) Determine the remainder when $x^4 - 3x^2 + 1$ is divided by $x + 2$.

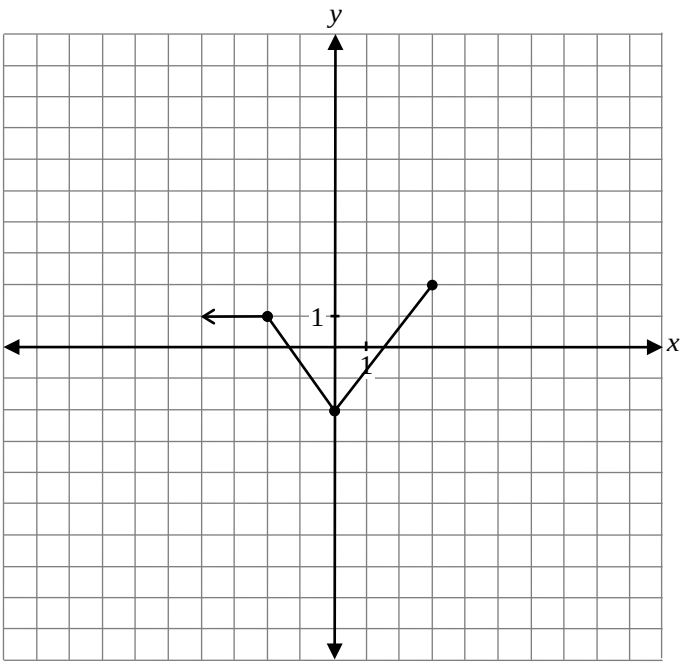
b) Is $x + 2$ a factor of $x^4 - 3x^2 + 1$?

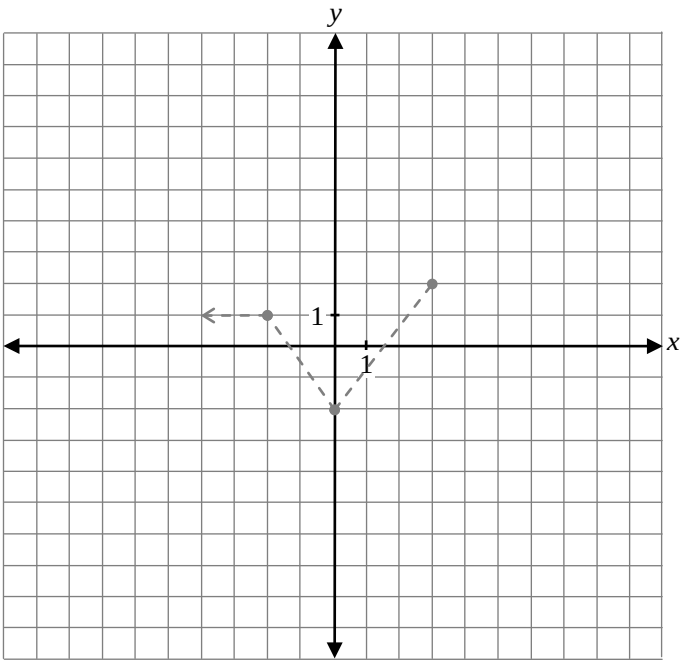
Explain your reasoning.

Question 9 — 2 marks

111

Given the graph of $y = f(x)$ below, sketch the graph of $y = 2f(x) - 3$.





The graph of $f(x)$ has already been drawn for your reference.

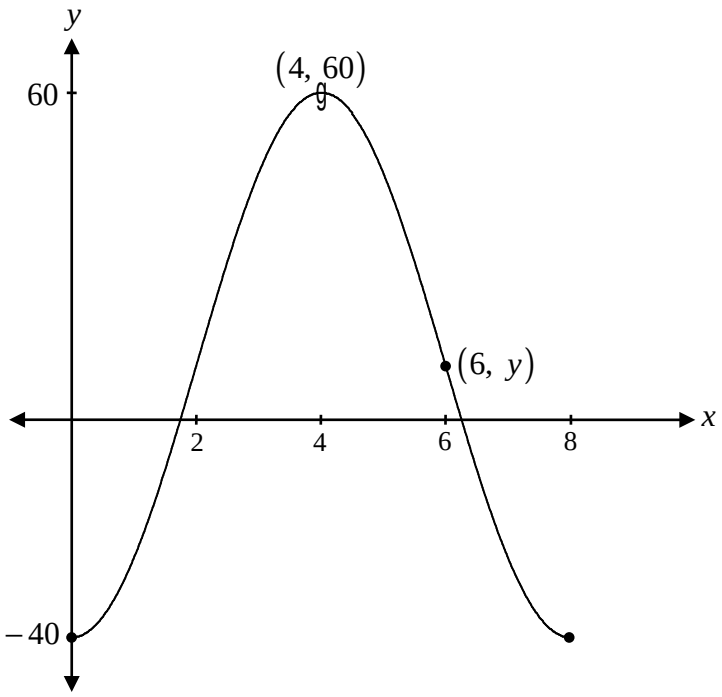
No marks will be awarded for the graph of $f(x)$.

Question 10 — 1 mark — 112

Determine one possible restriction for the domain of $f(x) = (x-1)^2$ so that the inverse of $f(x)$ is a function.

Question 11 — 1 mark — 113

Using the graph of the sinusoidal function below, find the value of y in the point $(6,\ y)$.

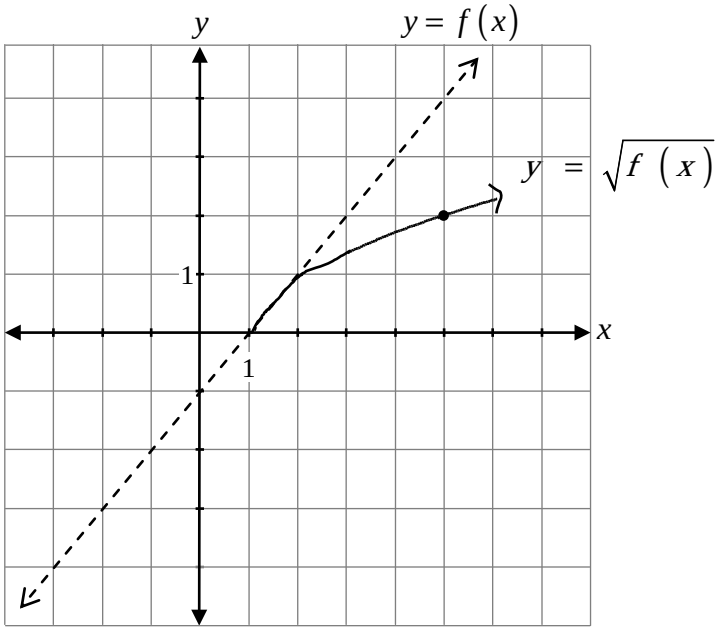


Question 12 — 1 mark — 114

Billy was given the graph of $y = f(x)$.

He was asked to sketch the graph of $y = \sqrt{f(x)}$.

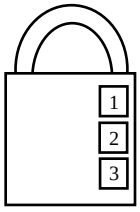
His answer is given on the graph below.



Explain the error Billy made when sketching the graph of $y = \sqrt{f(x)}$.

Question 13 — 1 mark — 115

Explain why a locker combination should really be called a locker permutation.



Question 14 — 1 mark — 116

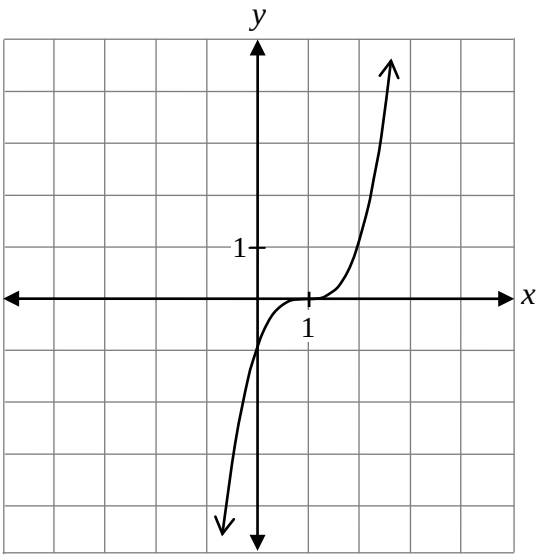
The graph of $f(x) = x^2 + 4$ is reflected over the *x*-axis.

Write the equation of the new function.

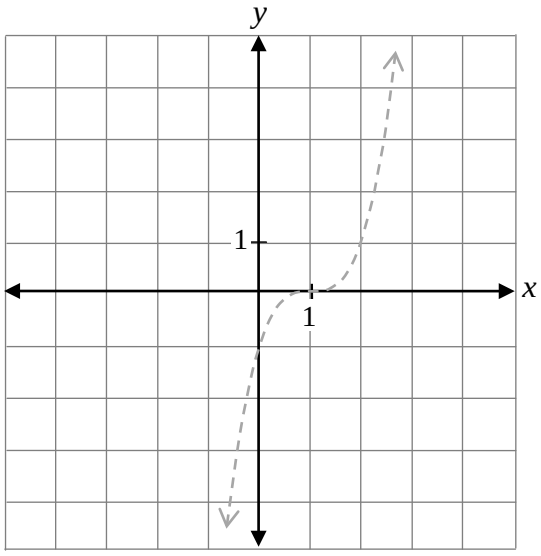
$y =$ _____

Question 15 — 2 marks — 117

Given the graph of $y = f(x)$ below,



sketch the graph of $y = \dfrac{1}{f(x)}$.



The graph of $f(x)$ has already been drawn for your reference.

No marks will be awarded for the graph of $f(x)$.

Question 16 — 2 marks — 118

Divide $\left(x^3 - 5x - 4\right)$ by $(x+1)$.

Question 17 — 1 mark — 119

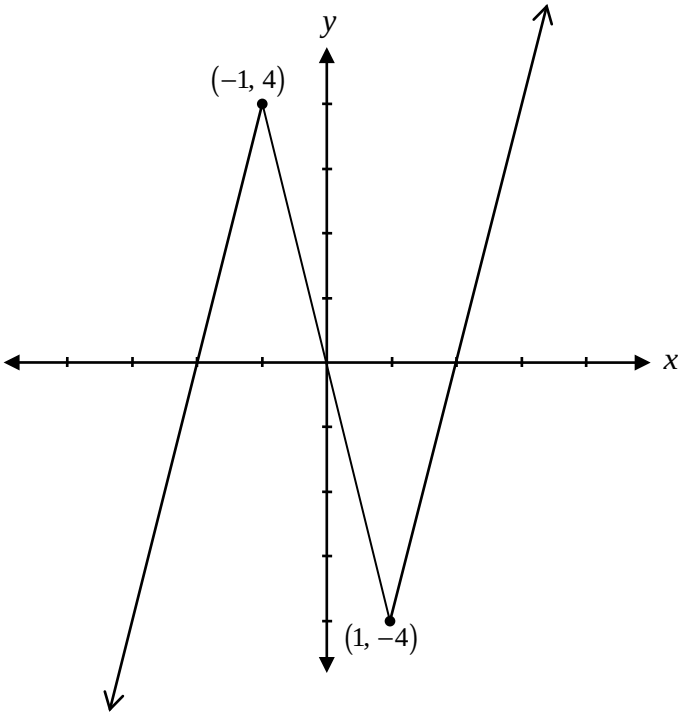
You are given the following row of Pascal's Triangle.

1 7 21 35 35 21 7 1

Determine the values of the next row.

Question 18 — 2 marks — 120

Given the graph of $y = f(x)$ below, state the domain and range of $y = \sqrt{f(x)}$.



Domain: _____

Range: _____

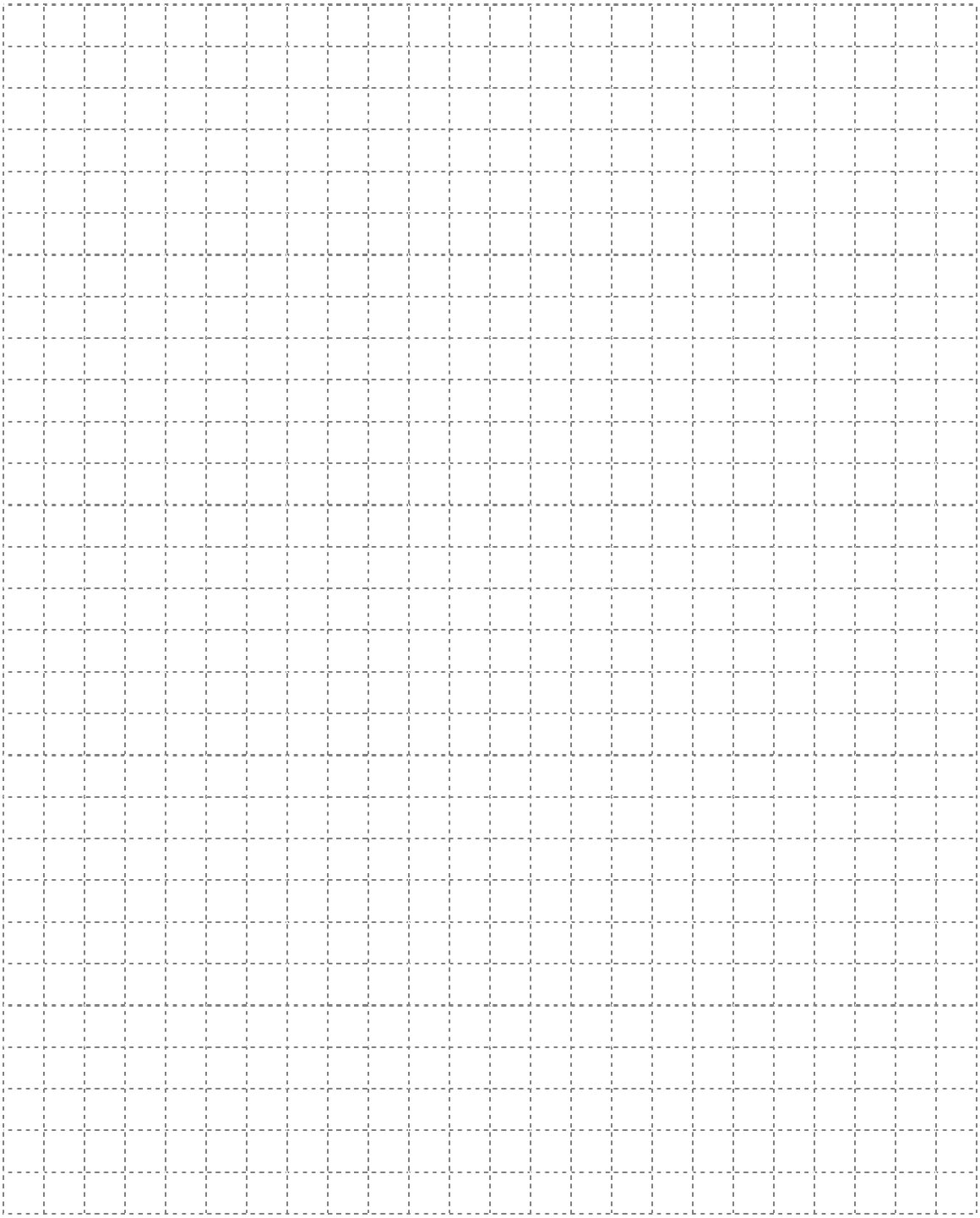
Question 19 — 3 marks — 121

Prove the identity below for all permissible values of θ:

$$\frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$$

Left-Hand Side	Right-Hand Side

No marks will be awarded for work done on this page.



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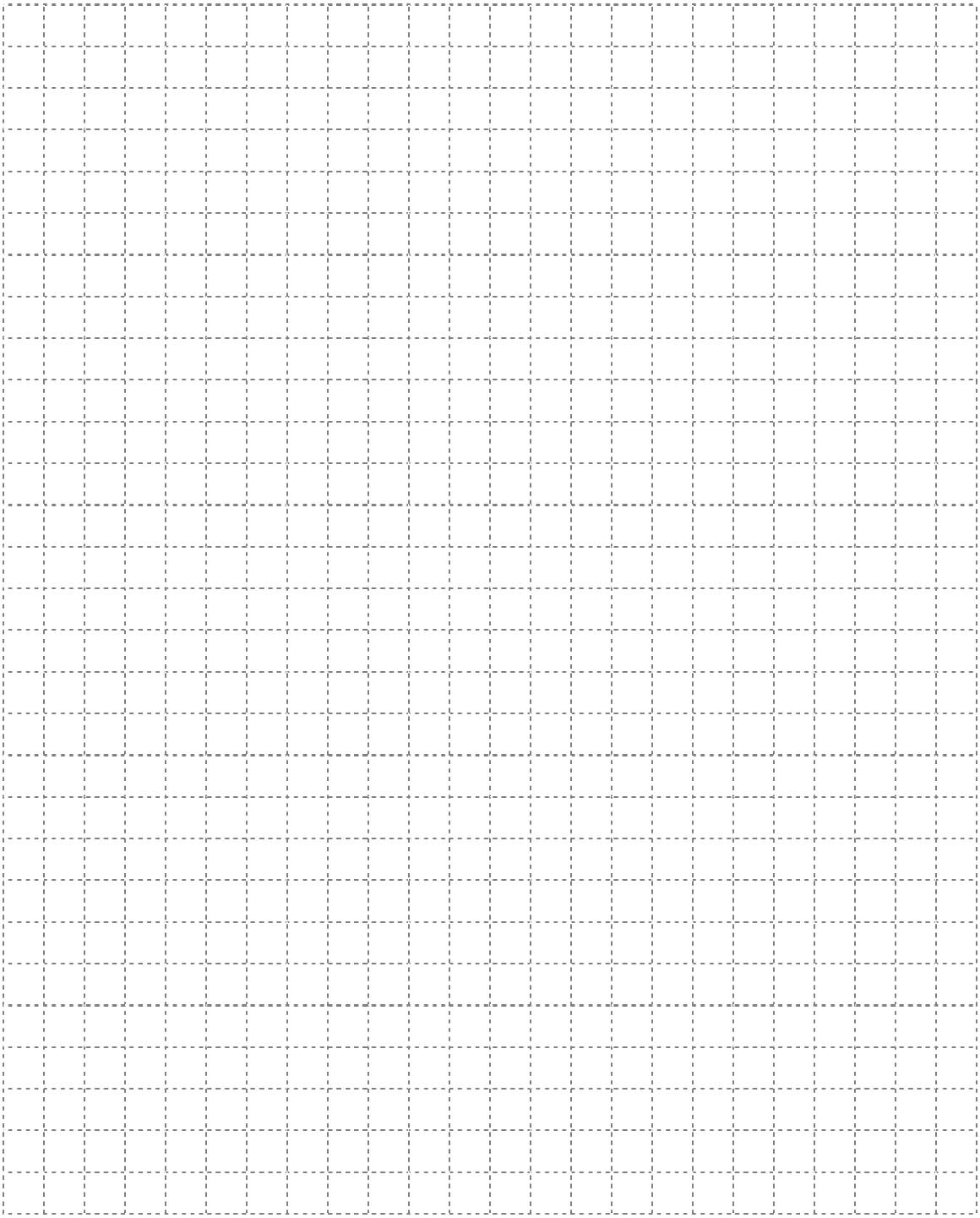

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